

The influence of molecular diffusivity on turbulent entrainment across a density interface

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The rate of mixing across a density interface between two layers of liquid has been measured in a laboratory experiment which allows a direct comparison between heat and salinity transports over the same range of density differences. Low Reynolds number turbulence was produced by stirring mechanically at a fixed distance from the interface, either in one or in both layers, and the results for these two sets of experiments are also compared. The measurements cover a factor of two in stirring rate and twenty in density. Over this range of conditions the ratio of entrainment velocity to stirring velocity can be expressed as functions of an overall Richardson number Ri , and in this form the results of the one and two stirred layer experiments are indistinguishable from one another. For density differences produced by heat alone, the functional dependence is close to Ri^{-1} except at small values of Ri where it approaches a finite limit. For experiments with a salinity difference across the interface, the mixing rate is the same as in the heat experiments at low values of Ri , but falls progressively below this as Ri is increased, with the approximate form $Ri^{-\frac{1}{2}}$.

An interpretation of these results has been attempted, using a dimensional analysis and qualitative mechanistic arguments about the nature of the motion. The Ri^{-1} dependence implies a rate of change of potential energy proportional to the rate of working by the stirrer. The decreased mixing rates for salt have been attributed to a slower rate of incorporation of an entrained element into its surroundings by diffusion, which increases the tendency for it to return to the interface and dissipate energy in wave-like motions.

1. Introduction

A basic step towards the understanding of vertical mixing in a stably stratified fluid is the study of the mixing processes at a single density interface. Even in situations where density distribution in the large is generally assumed to be a smooth, nearly constant or slowly varying gradient, there can be significant smaller scale variations about the mean. There is now a great deal of observational evidence which suggests that in the ocean, for example, it is typical to find many well-mixed layers separated by much sharper interfaces. (For a summary of some of the recent findings, see Cooper (1967) and Stommel & Federov (1967).) The details of these oceanic observations and the various mechanisms which

have been proposed to account for the formation of layers do not concern us here. It is clear, however, that such interfaces exist, and that once they are formed the vertical transports of properties like salt and heat will be dominated by these more stable regions.

The present paper describes a laboratory experiment which has been designed to investigate a previously neglected feature of the mixing across an interface. With the eventual application to the ocean in mind, the effect of molecular diffusivity on the entrainment rate will be studied by comparing directly the mixing rates across density steps produced by differences of temperature and salinity. By admitting the possibility of some molecular effect, one is already departing from a common line of approach to this problem, which is as follows. The idea of high Reynolds number similarity is applied to the case of stratified flow, and it is again assumed that the nature of the working fluid and the Reynolds number Re are not relevant parameters provided Re is large enough. Further, when effects attributable to molecular viscosity *are* observed in a laboratory model of a geophysical process, it is usually implied that the relevant results are those at the limit of large Re , i.e. that the measurement of 'fully turbulent entrainment' is the eventual aim. In a very stably stratified fluid, however, both the length scale in the vertical and the velocities can be reduced to such an extent that the Reynolds number of the turbulence becomes quite small. Even in the ocean it is sometimes appropriate to discuss mixing at such low values of Re that there is little fine structure on the energy-containing eddies of the turbulence. This is the case studied in the present experiments. The results should therefore also be relevant in a geophysical context, though they do represent a different limiting condition from the one usually considered.

It will be helpful to relate the present work to several different kinds of laboratory experiment which have been carried out in the past. First, there are the studies of turbulent entrainment in which mixing across a stable density interface is produced by a mechanical stirring on one side of the interface. An early experiment of this kind was conducted by Rouse & Dodu (1955), and similar work which had the well mixed layers near the surface of the ocean in mind has been reported by Cromwell (1960) and Turner & Kraus (1967). In each of these experiments the stirring was produced by the oscillation of a grid at a position parallel to and some distance from the interface, and the entrainment rate deduced either by observing directly the mean motion of the interface or by measuring the change in concentration of salt in the stirred layer. The present experimental technique is similar to this, with some important changes and additions.

Experiments using a stirring grid are certainly convenient for some purposes, but they can be criticized on the grounds that one is thereby imposing a length scale and a structure on the turbulence which depend on a rather special method of generation. More commonly, in nature, the turbulence is produced by shear, so that its structure is related to the geometry of the flow itself and not to any other length scales. This is so, for example, when a layer of heavy salt solution flows down a sloping bottom under a deep layer of fresh water at rest, as in the experiments of Ellison & Turner (1959). They showed that the ratio of the en-

traintment velocity to the mean flow velocity V is a function of a stability parameter having the form of a Richardson number $Ri = g(\rho - \rho_a)h/\rho_a V^2$, where ρ and ρ_a are the mean densities of the layer and the ambient fluid and h is the layer depth. This form of dependence of the mixing rate on the density difference implies that the property producing the density variations (whether it be heat, salt or suspended silt) has no other effect on the mixing. Since their laboratory results were applied to the case of methane mixing with air in mines, Ellison & Turner discussed the possible influence of molecular diffusivity in the light of a suggestion made by Batchelor & Townsend (1956), but it was rejected as an important factor, at least at high Reynolds number. Although this assumption appears to have been justified in the context of those experiments, which were conducted at fairly small values of Ri , the present work shows that it is *not* always possible to ignore molecular effects when the stability is much higher and the Reynolds number is not very large.

A third relevant type of experiment is one in which there is again no mean flow, but where the turbulence is produced by convective stirring. Turner (1965) examined the case of a two-layer system, stabilized with a salinity difference across a sharp interface but partly destabilized due to heating from below. The interface can persist because the rate of transfer of salt is much less than that for heat; both layers can become turbulent, with the mean values of salinity and temperature nearly uniform except near the interface. Measurements were made of the rates of transfer of salt and heat across such a boundary, and the results expressed as a function of the ratio of the contributions of salt and heat to the density difference. Both the heat flux and the ratio of suitably defined mean transfer coefficients K_S/K_T were found to decrease markedly as the density ratio $(\Delta\rho)_S/(\Delta\rho)_T$ increased, i.e. as the interface became more stable. K_S/K_T in fact ranged all the way from unity at $(\Delta\rho)_S/(\Delta\rho)_T = 1$ to nearly the molecular ratio $k_S/k_T \approx 10^{-2}$ at the highest stabilities studied, which implies that molecular processes were becoming progressively more important as the stability of the interface increased.

The experiments just described have provided the immediate incentive for the present work, since they raise again the question of the role of molecular diffusion. They were, however, more complicated than one would wish; not only are two properties contributing to the density difference and being transferred together across the interface, but the turbulence arises as a result of the flux of heat. In order to separate the stirring from the consequent transport, it therefore seemed useful to return to the experiments with mechanical stirring, i.e. the case of an entirely forced motion, and to investigate *separately* the transports of heat and salt across an interface for the same range of density differences and stirring rates. The main aim of this paper is to present this experimental comparison between the transports of properties with very different molecular diffusivities. Experiments have been carried out with stirring on both sides, or alternatively only on one side, of an interface, and the comparison of these two conditions also sheds some light on the nature of the mixing process.

2. Theoretical preliminaries

The main theoretical discussion and the attempts at interpretation will be left until after the experimental results have been presented, but there are several points which must be considered before the data can be organized in the most economical way.

The stirring motion

The comparisons which are to be made do not depend critically on the exact nature of the stirring motions, since care has been taken to keep both the geometry and the stirring the same from experiment to experiment. (See figure 1 and the appendix for the details of the apparatus and technique.) The form in which the data are to be plotted does, however, entail some assumption about the relation between the frequency of oscillation of the stirring grid and the velocities produced by it. There is some information available about the flow due to a long circular cylinder oscillated with small amplitude perpendicular to its length (see, for example, Schlichting 1955, p. 224) but little is known for other shapes or large amplitude oscillations. S. M. Thompson (personal communication) has carried out some experiments using single bars of various shapes which suggest that qualitatively the flow is much the same for different cross-sections; a *steady* jet-like mean flow is produced at some distance from the cylinder, which is outwards along the plane parallel to the direction of motion, and inwards along the perpendicular plane. The magnitude of the velocity for a fixed geometry is, to the accuracy attained, proportional to the frequency n .

With a number of such bars in a regular array one might expect that the individual outflowing jets will eventually break down and produce a turbulent motion whose length scale could depend on the detailed geometry, but whose velocity scale at a given distance from the grid will again be proportional to the frequency. The proper justification of this will require a further detailed investigation, and for the present we must regard it as a plausible assumption.

Specification of the mixing rate

It is convenient when comparing the rates of transfer of different properties across an interface to express them all in terms of an entrainment velocity u_e . When there is stirring only on one side of the interface, u_e is directly the mean rate of advance of the interface, but when both sides are turbulent and the interface is stationary, u_e must be related to the rate of change of properties in one of the layers. This latter method can also be used of course as a check on the direct method in the case of a single stirred layer.

Let the concentrations of salt in the top and bottom layers be c_1 and c_2 respectively, and the layers be of equal fixed depths h . The entrainment velocity u_e can always be defined as the rate of mixing in one direction (say from top to bottom) across the interface, whether or not the upper layer is stirred, and is given by

$$u_e = \frac{-h}{c_2 - c_1} \frac{dc_2}{dt}. \quad (1)$$

Thus u_e is uniquely defined by the layer depth, the concentration (or temperature) difference, and the rate of change of concentration of the layer *into* which mixing is taking place. The total rate of change of the concentration difference

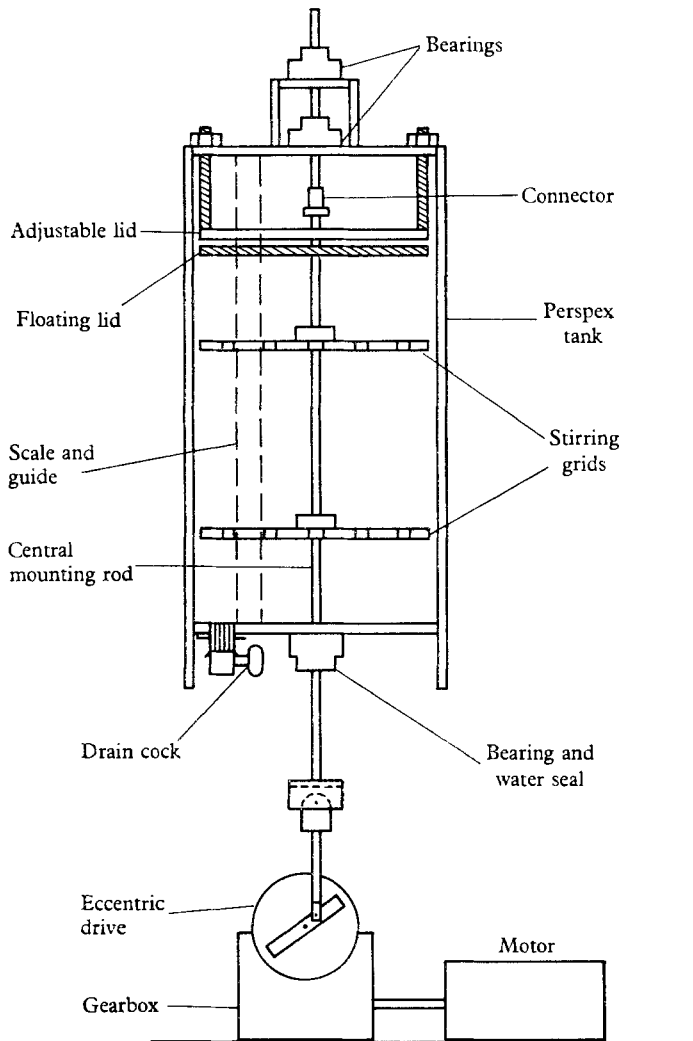


FIGURE 1. A sketch of the experimental tank.

will of course be different if there is stirring above as well (as will be seen later, it is just doubled if the stirring is symmetrical, since u_e is unchanged). The above definition of u_e continues to hold, but it may now also be expressed in terms of the rate of change of concentration in the upper layer,

$$u_e = \frac{h}{c_2 - c_1} \frac{dc_1}{dt}. \tag{1a}$$

Dimensionless parameters

With the stirring and mixing specified through n and u_e as just described, appropriate dimensionless parameters can now be chosen. It will be assumed, following Rouse & Dodu (1955), that the geometry can be specified by a single length scale l , though other geometrical ratios will be implicit in the results. The density difference, multiplied by g , will also be a basic variable, and it is natural to choose (as did Ellison & Turner (1959)) the main independent dimensionless ratio to have the form of an 'overall Richardson number'

$$Ri = g \frac{\Delta\rho}{\rho} \frac{1}{ln^2}. \quad (2)$$

In the figures which follow, the ratio of u_e to a characteristic stirring velocity nl has in effect been plotted against Ri . Since l is fixed (but unknown) the actual plots are of u_e/n (with units cm/cycle) against $\Delta\rho/n^2$ on an arbitrary scale.

A fuller discussion of the choice of a parameter which can best describe the molecular effects will be given after the form of the experimental results has been displayed. There is clearly a wide choice *a priori* of possible dimensionless ratios involving the kinematic viscosity ν and the diffusivity κ (for salt or heat) in addition to the quantities already used. It will appear, however, that a plausible interpretation can be made using only Ri and a Peclét number $Pe \propto nl^2/\kappa$. In contrast to various other suggestions made on experimental and theoretical grounds, it does not seem to be essential here to use ν , either in the form of a Reynolds number nl^2/ν or a Prandtl number ν/κ .

3. Experiments with one layer stirred*Qualitative description*

When only one layer was stirred, this was always the bottom one; the top grid was removed, but otherwise the geometry was kept the same as that described in the appendix. The tank was filled by putting the lighter fluid in first, and carefully pouring the heavier layer under this until the interface was just below the standard level. Stirring was begun at the desired rate, and the interface allowed to rise until it was judged to be at the level of the central mark; it was then kept there by withdrawing fluid from the bottom layer at such a rate as to produce a downward velocity u_e , which compensated exactly for the entrainment and was directly measurable. The depth of the upper layer was therefore always decreasing, but the experiments were stopped before the floating lid could affect the motion near the interface. In the experiments with temperature differences, the change in temperature of the bottom layer was used as a second measure of the entrainment.

The major source of error in the direct measurement of entrainment arises because of the subjective judgement of interface position; with an interface at the wrong distance from the stirring grid the same stirring frequency will produce a different velocity near the interface. This is particularly true when the

density differences are small and large excursions of the interface are possible. The typical appearance of such an interface, with the stirred layer dyed, is shown in figure 2*a* (plate 1). Note the large eddy structure, with a sharp (but distorted) boundary advancing into the fluid at rest. The difficulties arise in making, visually, a space average over this boundary and adjusting the outflow rate to keep it at the desired level, not because of any uncertainty about the definition of the instantaneous edge, which always appeared sharp. At the end of an experiment when both stirring and the withdrawal are stopped, one can compare the stationary interface with the height and thereby estimate the errors: these were up to $\frac{1}{2}$ cm for the most distorted interfaces.

It is revealing to compare 2*a* with another photograph (figure 2*b*, plate 1) taken of a mixing interface under similar conditions of stirring and density difference (produced with salinity), but with dye now added only to the *unstirred* layer. This appears quite different, and shows how the large eddies seen in 2*a* are entraining fluid in the form of sheets, which are ultimately mixed rapidly through the whole of the stirred layer. Molecular diffusion out of these sheets will play an important part in the explanation given in §5 of the differences between the experiments with heat and salt.

Quantitative results

The direct measurements of entrainment velocity for all the experiments in which only the bottom layer was stirred are plotted in figure 3, using the form of presentation suggested in §2. The scales are logarithmic, with the mixing rate (in units of cm/cycle) as the ordinate, and the stability parameter (proportional to a Richardson number Ri defined by equation (2)) along the abscissa. The maximum value of $\Delta\rho/\rho$ used was about 5×10^{-3} , and the stirring frequency was varied in the range 166–333 rev/min. Since the geometry was fixed, but the appropriate length scale l is not known, the scale shown is an arbitrary multiple of $\Delta\rho/\rho n^2$. (It is in fact $3 \times 10^8 \Delta\rho/\rho n^2$, where $\Delta\rho/\rho$ is expressed as a fraction and n is the stirring speed measured in cycles per minute.) It will be suggested in §6 how a more definite meaning can be given to the stability parameter Ri .

The solid points in figure 3 represent the experiments with temperature differences alone. Over most of the range of stabilities covered, these results can be represented fairly well by a line of slope -1 . This implies the functional form

$$u_e/n \propto Ri^{-1}, \quad (3)$$

or alternatively, that u_e is inversely proportional to the temperature difference at a given stirring rate.

There are two consequences of these results which are so simply derived that they will be mentioned now. First, it follows from (1) that with fixed stirring, the rate of change of temperature of the bottom layer is independent of the temperature difference: an increase in u_e is very nearly compensated by the reduction in the heat transported by the fluid which is mixed. (A closer examination of the data reveals a systematic dependence of heat flux on the temperature difference; this is small in the range covered by our experiments and will be neglected in the present context, but it has been the subject of careful study

by C. G. H. Rooth (personal communication.) Secondly, (2) and (3) imply a rate of change of potential energy of the stirred layer which is proportional to the rate of production of turbulent energy by the grid. This was first pointed out by Rouse & Dodu (1955), though their own experiments (using salt) did not support it. The implications of this result will be further examined in §5.

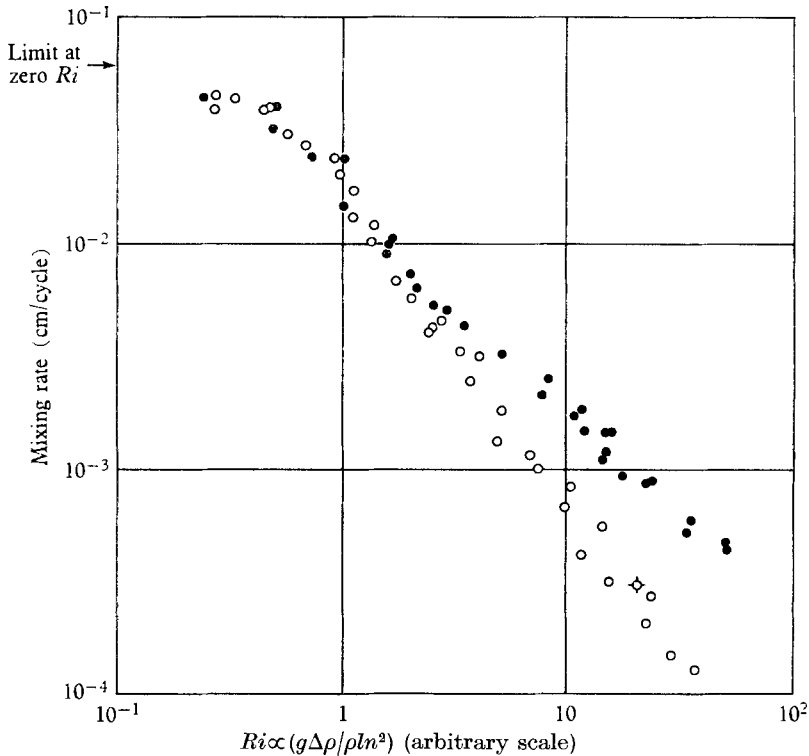


FIGURE 3. The comparison between directly measured entrainment velocities with stirring on one side of the interface. The symbols denote density differences produced with: ●, heat; ○, salt. The experiment marked \diamond was carried out with both layers 15°C hotter than normal. Mixing rates are plotted against a stability parameter having the form of a Richardson number, on logarithmic scales.

At the lower values of Ri used, the mixing rate deviated from the Ri^{-1} law, and approached the limit of mixing rate appropriate to zero Ri , i.e. zero temperature difference. This limit was established from a separate series of experiments in which a turbulent front, marked with a little dye but with no temperature difference, was held at a steady height by withdrawing fluid at the appropriate rate. This is a useful reference which includes the effects of all the geometrical parameters inherent in the rest of the experiments, and which can be used to make comparisons between different kinds of experiments, as suggested in §6.

The open circles on figure 3 show the results obtained when salinity differences were used to produce the density step. At small values of Ri there is no significant difference between the mixing rates with salinity and temperature differences, and the points approach the same limit as the density step tends to zero. At

higher values of Ri , the salinity results begin to diverge from those obtained with temperature differences, and are now best represented by $u_e/n \propto Ri^{-\frac{1}{2}}$. The mixing rate with salinity has fallen to only a fifth of that with temperature at the highest value of Ri achieved in these experiments. This experimental demonstration of a large difference between the mixing rates with different molecular diffusivities is the main result of this paper.

4. Experiments with both layers stirred

Qualitative observations

With both stirring grids in place, the tank was filled as before but with the total depth now adjusted to 36 cm. When stirring was begun, the interface was sharpened from *both* sides, but the rate of entrainment on the side of the interface closer to a stirring grid was greater, so the interface gradually moved to a central position. The quantitative measurements were not begun until this symmetrical state had been achieved. This qualitative fact, already implied in the calculation of §2, is worth emphasizing: not only does a density interface stirred from one side remain sharp, but it is also possible to maintain a sharp *stationary* boundary in a two-layer system, most of which is kept turbulent by mechanical or convective stirring, provided there is the same intensity of turbulent motion on each side of the interface.

Quantitative results

A direct measure of mixing rates is now not available, but equivalent values of u_e have been deduced from the temperature and salinity changes as outlined in §2. Both upper and lower layer temperatures were recorded, and corrections were applied for the heat lost through the walls. The results with temperature differences are shown as crosses in figure 4; the scales have exactly the same meaning as in figure 3 (except that the experiments do not extend to such low values of Ri as before).

Salinity, on the other hand, was measured only in the top layer, and the entrainment velocity deduced using equation (1*a*). The total amount of salt added initially was weighed, and must be conserved, and so both the flux and the density difference could be obtained from a sequence of measurements of this single quantity. The salinity results are shown as squares on figure 4. Over the smaller range of stability conditions covered, figure 4 looks very like figure 3; again the entrainment velocity is a systematic function of Ri , and the salinity curve falls below that obtained with temperature differences.

A direct comparison of all the experimental runs with temperature differences and stirring on one or on both sides of the interface is made in figure 5. The points have the same meaning as in the earlier figures, but in addition the indirect measure of mixing obtained from the (corrected) temperature rise in the case of single stirring has been added. In order to assess the reality of the differences, the systematic error in mixing rate due to a change in grid-interface distance of ± 0.5 cm from the nominal 9.0 cm was estimated in a separate series of experiments (with two grids). The error amounts to about 30 %, and this is enough

to account for the greater differences at the smaller values of Ri , where the adjustment of level was more difficult. To within the accuracy with which the geometry can be held fixed, the rates of mixing in the single and double stirred experiments are therefore indistinguishable from one another, as are the two estimates of mixing velocity in the case of stirring on one side of the interface. All of them support the approximate form of dependence (3) on Ri .

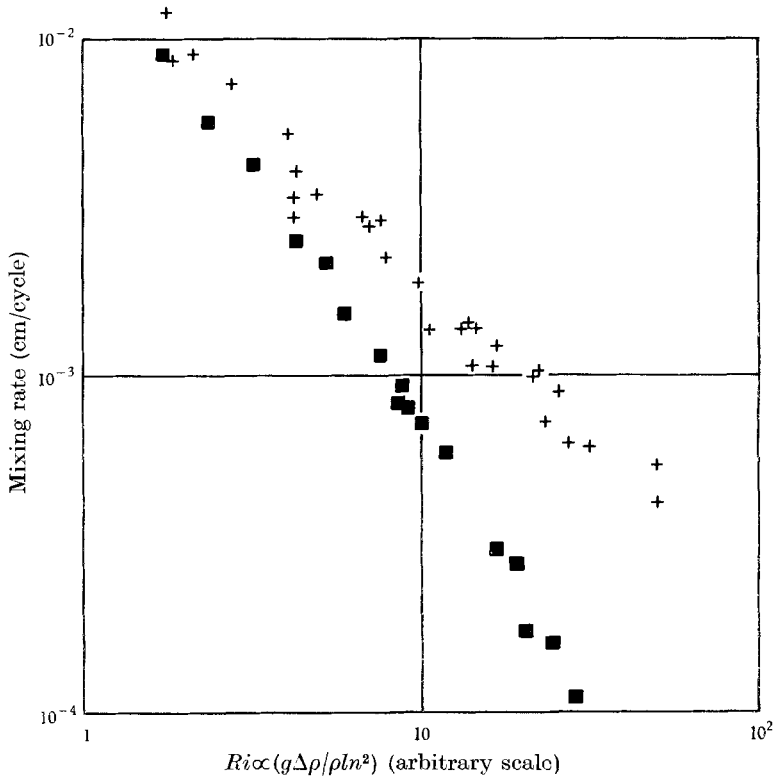


FIGURE 4. The entrainment velocities deduced from temperature and salinity measurements, with stirring on both sides of the interface. Density differences are due to: +, heat; ■, salt.

Similarly, all the experiments with salinity differences are compared in figure 6. In this case only the two kinds of measurements already described were available, and have been transferred from figures 3 and 4. Again there is no significant difference between the entrainment rates with stirring on one side or both sides of the interface, and all the measurements are described by $u_e/n \propto Ri^{-\frac{3}{2}}$ over this range of Ri .

5. Interpretation of the experiments

The development of a satisfactory detailed theory of the phenomena described remains a major task, and far less than this will be attempted here. Instead, the parameters which could account for the differences observed will be discussed,

and various heuristic arguments will be put forward in the hope of shedding some light on the physical processes involved, and especially on the role of molecular diffusion which has been left out of account entirely in previous work.

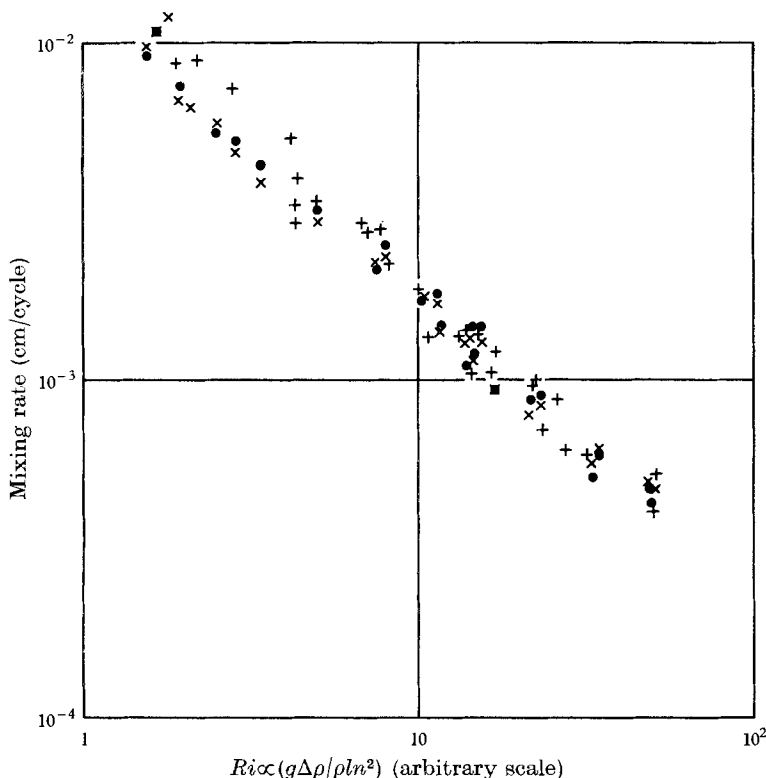


FIGURE 5. The comparison between three types of experiments in which temperature differences were used to produce density differences across an interface. The symbols denote: ●, stirring on one side of the interface, velocity measured directly; x, stirring on one side of interface, velocity deduced from temperature; +, stirring on both sides of interface, velocity deduced from temperature.

The effect of viscosity

It is immediately clear that the observed experimental differences between mixing rates with heat and salt cannot be explained by differences in viscosity (or Reynolds number). The geometry was fixed, the velocity was changed by a factor of two, and over the range of temperature and salinity used the kinematic viscosity changed by a factor of only 1.5; comparisons were, moreover, made between the two properties at essentially the same values of Re and Ri . (The unimportance of the viscosity changes due to temperature was confirmed directly by the run made with both layers at a higher temperature, shown in figure 3.) Though the interpretation to be given here will depend only on the fact that the Reynolds number of the turbulence should be small, this parameter was invoked explicitly by Rouse & Dodu (1955) to explain their experimental results, and so some discussion of the point is desirable here.

The use of the stirring frequency as a parameter to define Ri (as in §2) only

makes sense physically if this is a valid measure of the fluid velocities *near the interface*. The turbulent velocities will certainly decay with increasing distance away from the grid, as has been observed, but the assumption that the velocity

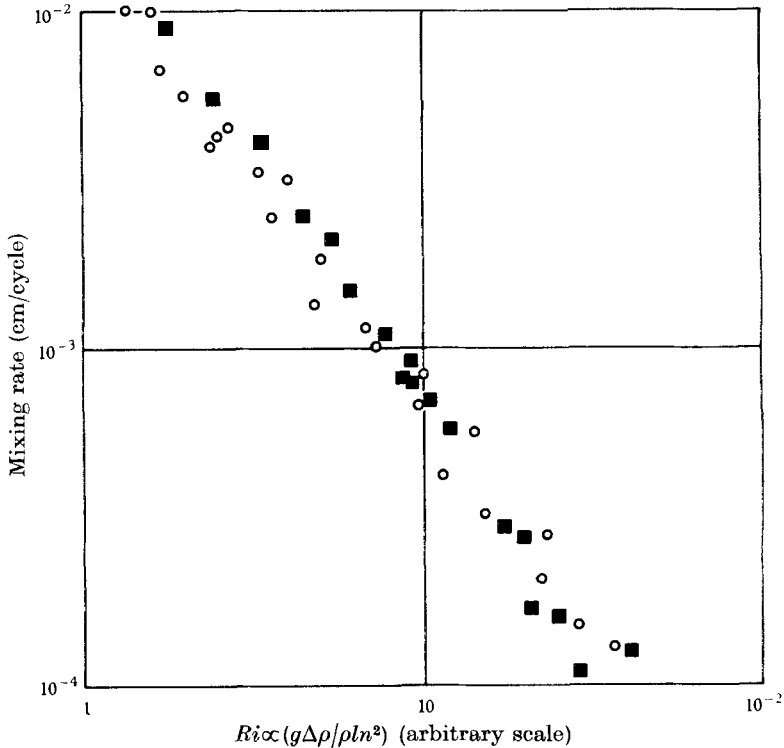


FIGURE 6. The comparison between experiments in which salinity was used to produce density differences across an interface: \circ , stirring on one side of interface, velocity measured directly; \blacksquare , stirring on both sides of interface, velocity deduced from salinity.

scale everywhere is proportional to the grid velocity u_g implies that the grid frequency cannot enter strongly in any other way into the decay law. Both geometry and viscosity could be relevant in a discussion of the production and decay, but it seems most unlikely that their effects can be expressed in terms of a function of a Reynolds number hu_g/ν based on layer depth, as suggested by Rouse & Dodu. This form of dependence implies that an increase in h would have the same effect as an increase in u_g or a decrease in ν , which is physically unrealistic.

The problem is far from solved, but for the present purposes it is only necessary to emphasize that three separate processes are involved when stirring is produced by a grid: the generation of the motion near the grid, the decay of the turbulence between the grid and the interface, and the mixing across the interface itself. The first probably depends on viscosity, and could explain the differences in experiments where ν is systematically varied, but the others need not. If the velocity and length scale of the energy containing eddies are assumed to be specified near the interface, the present experiments can be interpreted without a further appeal to viscous effects.

Molecular diffusivity

The major parameter determining the mixing rate is clearly Ri , but it remains to be explained how there can be a *different* functional dependence on Ri when the molecular diffusivity is changed. Is either form to be regarded as a standard or ideal case to which deviations can be referred? The simplicity of the result (3), and its consistency with the energy argument, encourages the view that in the experiments with temperature differences some kind of limiting state was approached where only the energy balance mattered and diffusivity did not enter the problem directly. This will be taken as the working hypothesis here, and reasons for a decreased mixing rate with salinity differences will be examined.

A qualitative picture of a process leading to the observed kind of dependence on diffusivity is the following. Suppose that an element of heavy fluid in the form of a sheet (see figure 2*b*) is lifted out of the interface by a large eddy of the stirring motion, an action which involves a certain amount of mechanical work. If the velocity of ejection and the molecular diffusivity are high enough, this element becomes indistinguishable from its surroundings before it can fall back, and all the work done in lifting it appears as potential energy. This is the case for heat, and so an energy argument is applicable. For the same velocity of ejection and a much lower diffusivity, on the other hand, a heavy sheet of fluid can fall back to the interface before it has diffused into its surroundings. For salt, therefore, only part of the work done remains permanently as potential energy, and the rest is dissipated in wave-like motions near the interface. It is clear that the limitation of small Reynolds number turbulence (at least near the interface) is essential to this picture, to ensure that the diffusion will be dominated by molecular effects and not by mixing due to small-scale turbulent motions. A quantitative mechanistic model can be constructed along these lines, but it is perhaps more informative to develop the idea using a less restrictive dimensional argument.

The given parameters, in terms of which the entrainment velocity u_e is to be expressed, are now the velocity scale u_1 and the length scale l_1 , say, of the energy containing eddies near the interface, the buoyancy parameter $g\Delta\rho/\rho$ and the molecular diffusivity κ . With the introduction of an extra parameter there must now be a functional relationship between three non-dimensional groups. It is useful first to express these as ratios of four velocities: u_e , u_1 , a diffusion velocity u_d and a 'buoyant velocity' u_b defined by

$$u_d = (\kappa/t_1)^{\frac{1}{2}} \quad \text{and} \quad u_b = \{g(\Delta\rho/\rho)l_1\}^{\frac{1}{2}}, \quad (4)$$

where t_1 is a time scale for the diffusion process. Two of the dimensionless groups have already been chosen to be u_e/u_1 and $Ri = (u_b/u_1)^2$, using the new notation. There are in principle two choices for the third if the dependent variable u_e is not to be used again. If t_1 is identified with the time scale of the large eddies l_1/u_1 , then the first possibility is

$$(u_1/u_d)^2 = u_1 l_1 / \kappa = Pe, \quad (5)$$

a ratio having the form of a Peclet number. The other is

$$(u_b/u_d)^2 = Ri Pe = g\Delta\rho l_1^2 / \rho \kappa u_1. \quad (6)$$

If it is assumed that a relation of the form (3) is to be modified by multiplying by some function of this extra ratio, then clearly (5) is not the proper ratio to use, since this would imply just a displacement of the mixing rate for salt relative to heat over the whole range of Ri . (For the same reason a dependence on a Prandtl number ν/κ is ruled out, even if ν were admitted as a relevant parameter.) The functional form which follows when (6) is used,

$$u_e/u_1 = Ri^{-1}f(RiPe), \quad (7)$$

does have all the features needed to explain the experimental results, and is consistent with the model described qualitatively above. By its definition (6) $RiPe$ expresses the balance between buoyancy, which is tending to return an element of fluid to the interface, and diffusion, which causes it to merge with the turbulent layer. It becomes small (and unimportant) when either Ri is small or κ is large. The reason for the two different forms of dependence on Ri alone is just that the major change in Pe is produced by a change in κ and therefore in diffusing substance. The additional change due to the variations in u_1 over the range of these experiments is small, and so Pe effectively has two different constant values in the experiments with temperature or salinity differences.

An empirical function which describes most of the range of the experimental results, and in particular the fact that the extra diffusive term only becomes relevant above a certain value of $RiPe$, is

$$u_e/u_1 = Ri^{-1}(c + (RiPe))^{-\frac{1}{2}}, \quad (8)$$

where c is a constant. Although the absolute length and velocity scales are not yet known, the qualitative behaviour is right. When κ is large the extra term disappears, and when κ is small

$$u_e/u_1 \propto Ri^{-\frac{3}{2}}Pe^{-\frac{1}{2}}. \quad (9)$$

Nothing has been said about the transition between the buoyancy controlled régime and that with zero density difference. The position of this transition could be determined by a relation between a Reynolds number and Ri (as suggested by Phillips (1966)), but a test of his hypothesis lies outside the scope of the present experiments. It is unfortunate that the Ri^{-1} dependence is established at about the same place as the salinity and temperature curves diverge. If the interpretation suggested here is valid, this must be coincidental, but the point can only be properly resolved by further experiments designed to separate the two effects.

Only the Ri dependence in (9) has been tested experimentally; the $Pe^{-\frac{1}{2}}$ term has been suggested entirely on the basis of the dimensional argument. It is therefore of some interest to refer briefly to a different kind of experiment which leads directly to the same form of dependence on Pe , and which might also give some clues about the molecular processes near the interface. Chemical engineers have made measurements of the rate of absorption of a gas across a free surface into a stirred liquid, or a solute between two immiscible solvents, and these can be thought of as the limiting case of very high Ri . A recent study by Fortescue & Pearson (1967) of CO_2 absorption into water is based on the hypothesis of molecular diffusion into the large eddies of the turbulence. It predicts that the

mean transfer coefficient, which has the dimensions of velocity, varies as $u_1 Pe^{-\frac{1}{2}}$ (using the notation of this section), in good agreement with their experimental results.

Comparison between single and double stirring

No interpretation has yet been given of the fact that the entrainment velocity (in one direction) remains unchanged whether or not there is stirring on the other side of the interface. This can be so because not only is the turbulence very strongly damped by the region of high gradient, but also the events which cause the removal of fluid elements from the interface are so rare that the two sides can be regarded as statistically independent. That is, mixing takes place through a mechanism rather like the intermittent breaking of steep forced internal waves, which occur over only a small fraction of the area at any one time. (Figure 2*b* may perhaps give a false impression of the frequency, since it does not distinguish between various positions through the depth of the tank, and has, moreover, been selected to show a time of maximum activity.) The chance that the same part of the interface will be affected simultaneously by the stirring motions on both sides is very small.

The whole discussion has been focused on the stirred layers, and on the behaviour of the fluid elements ejected into them from the interface, and little has been said about the interfacial region itself. It is clear even from visual observations, however, that the structure through the interface must depend on a balance between molecular diffusion which tends to spread it out and mechanical stirring which sharpens it; the sharpness increases markedly as the stirring rate is increased. This raises a whole new set of interesting questions about the influence of molecular diffusion on the mean properties of the interface, and the relation between this structure and the mechanism and frequency of ejection. Again one must say that more detailed experiments will be needed to explore these questions, but they must be answered before one can claim to have a completely satisfying explanation of even the 'overall' mixing rate between two stirred layers of different density.

6. Application of the results

The arguments of the previous section have led to a consistent physical interpretation of the experimental results, but one which is still essentially qualitative. In the absence of a direct measure of turbulent velocities and scales near the interface, it seemed preferable to emphasize the direct comparison between the experiments with heat and salt, rather than attempting to express either of them in absolute terms. In conclusion, however, this question of scaling, and the quantitative comparison with experiments (or larger scale natural phenomena) in which the turbulence is produced in quite a different way, will be considered briefly. For some purposes it may be useful to have even order of magnitude estimates of the important parameters in a form which allows such comparisons to be made.

A plausible approach follows immediately from the model of the mixing process developed above. This suggests that one should in each case define a Richardson

number R_1 say, which is based on the length and velocity scales of the large eddies (rather than on some other 'overall' scale which may be less relevant near the interface). Sometimes, but not always, it will be possible to estimate R_1 simply from the known parameters, or to relate it to a stability parameter defined in another way. In the present experiments, the one absolute measure of velocity near the interface which is available is the entrainment velocity at zero density difference. With the standard geometry used, this was 0.25 cm s^{-1} at a stirring rate of 250 rev/min, and by analogy with the entrainment into a jet, one would expect the r.m.s. turbulent velocity producing the entrainment to be several times as great, say $u_1 = 1 \text{ cm s}^{-1}$. The best estimate of the scale l_1 of the large eddies is judged to be the grid spacing, $l_1 = 5 \text{ cm}$. Using these definitions and numerical values of l_1 and u_1 , with the measured density differences, the resulting value of R_1 is of order unity when the entrainment ratio has fallen to 0.2 times its value at $\Delta\rho = 0$ (about the point where the heat and salt curves diverge from one another in figure 3). The scales used in figures 3–6, which up to now have been treated as arbitrary, in fact have approximately the right absolute magnitude when R_1 is used as the stability parameter.

As an example of a flow experiment with which one might wish to compare the present work, consider the results of Ellison & Turner (1959). These were originally plotted in the form of an entrainment ratio against an 'overall' Ri based on the mean layer depth and velocity, but they can be reinterpreted in terms of R_1 if one is able to make an additional assumption. Suppose that the direct effect of the density gradients on the structure of the turbulence within the layer is small. This condition is very nearly satisfied in our 'stirred box' experiments where the velocity is imposed mechanically, but it must be examined more closely for a density current where the density differences are themselves producing the shear. If it is so, the large eddy scales will be constant fractions of the 'overall' scales, and using R_1 instead of Ri will be equivalent to shifting the u_e/u_1 vs. Ri curve along both axes on a logarithmic plot. The shape of Ellison & Turner's entrainment curve (and also the absolute magnitudes) can indeed be fitted to the present results up to $R_1 \approx 1$ when plausible values are used for u_1 and l_1 . At larger values of R_1 , however, the assumption that the turbulent structure is not affected by the density distribution becomes questionable. If the same kind of scaling is applied to the results of Lofquist (1960) (who investigated a flowing saline layer under fresh water at higher values of Ri), his u_e/u_1 curve is found to fall more steeply with increasing R_1 than it does in the present experiments. One interpretation is that the relations between the overall velocity and length scales, and those describing the motion near the interface, are being affected by the density gradient in this case. Comparisons with such experiments in terms of R_1 will only be possible if u_1 and l_1 can be measured directly.

Finally, estimates of the Peclet number in our experiments can also be written down, based again on the velocity and length scales of the large eddies. Using the molecular diffusivity for heat, $Pe = u_1 l_1 / \kappa_H \approx 5 \times 10^3$, and for salt, $Pe \approx 5 \times 10^5$. In the 'linear' range of the experimental results, the product $Ri Pe$ lies between 5×10^3 and 2×10^5 for the experiments with temperature differences, and between 5×10^5 and 2×10^7 for those with salinity differences. These ranges do not

overlap, so equation (8) is consistent with the experimental results, and molecular diffusion becomes important in reducing the entrainment when $Ri Pe$ is greater than about 5×10^5 . It must be emphasized again that this result, in common with most others in this paper, will only be valid if the Reynolds number of the turbulence is small enough for molecular diffusion, and not small-scale mixing, to dominate the process of incorporation of an entrained fluid parcel into its new surroundings.

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Appendix. The experimental method

All the experiments described in this paper were carried out in a Perspex tank, 25.4 cm square in cross-section and 40 cm deep, which is sketched in figure 1. Stirring was produced using grids of 1 cm square Perspex strips precisely made up into a 5×5 array with 5 cm between centres and 2.5 cm overhang (and with therefore about 2 mm clearance at the edges of the tank). These grids were mounted on a central stainless-steel spindle by means of a small boss and grub screw, and were prevented from rotating by a Perspex scale (which also served as the depth scale) fastened to one wall of the tank and passing snugly between two of the grid bars. The spindle was held at the top by two nylon bearings, which served to reduce the bending, and at the bottom by a third; this was found to be also a very satisfactory water seal. The grids were therefore tightly constrained to move only with a reciprocating motion in the vertical.

This motion was provided by an electric motor, gearbox and eccentric drive, operating through a connecting rod on the bottom of the mounting spindle. Both the speed and the amplitude could easily be varied, although the total stroke was in fact kept fixed at 1.0 cm throughout these experiments. The speed was changed by regulating the voltage supplied to the motor, and adjusted to a number of discrete frequencies by comparing it against the 50 cycle mains using a stroboscopic disk mounted on the driving wheel.

The top of the tank was closed by a Perspex lid whose vertical position could be adjusted. The lid was provided with small holes, through which thermometers and other measuring instruments could be inserted into the tank. Below the fixed lid was a second, made of 1 cm thick expanded polystyrene sheet, which floated on the water surface and formed an insulating, rigid top boundary whatever the level of the water. Throughout the experiments described here, the layer depths were both 18.0 cm, with the central positions of the grids half way between the solid boundary and the interface over which mixing was occurring, i.e. at 9 and 27 cm above the floor of the tank. The interface position was marked with a fine line drawn round the four sides of the tank.

Temperatures were measured with two 0–40°C bulb thermometers, graduated

in 0.1°C but estimated to 0.02°C . These were set up at the centres of the two layers, in one of the spaces of the stirring grid. The salinity was measured using a commercially available conductivity probe and bridge, which was calibrated directly against solutions of known concentration. The probe was inserted about 5 cm into the top layer when a reading was required, and withdrawn between readings. The temperature was recorded at the same time, and corrections made for temperature variation when necessary. All the measured quantities were thus read off directly and written down at discrete time intervals, rather than being recorded continuously. The densities of the two layers were not measured directly; they were deduced from the temperature and salinity using the values tabulated in the 'International Critical Tables'.

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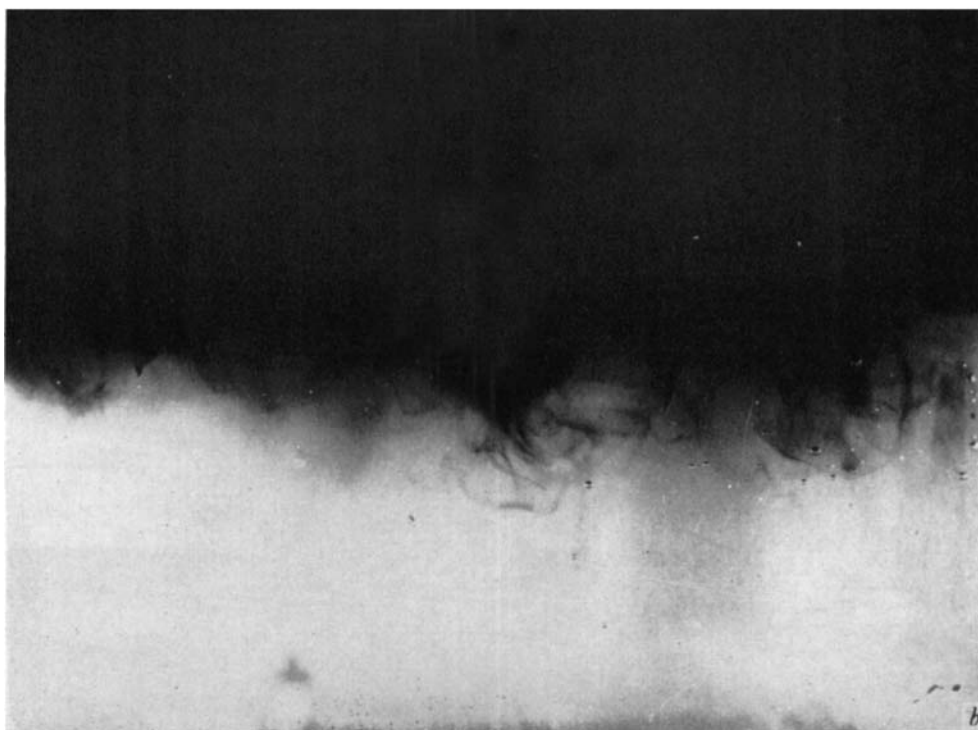
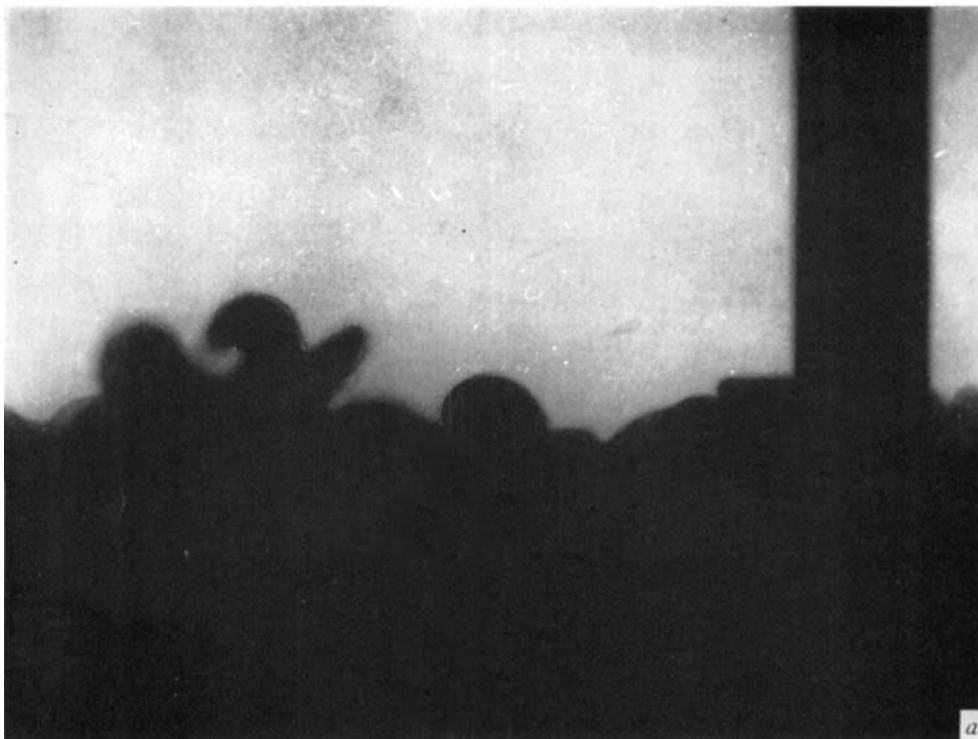


FIGURE 2. Photographs of a stable interface between salt and fresh water, with stirring on one side of the interface. Dye has been added to (a) the stirred (lower) layer, and (b) the stationary layer. A convenient scale is the support rod in (a) which is 6 mm in diameter.
TURNER (Facing p. 656)